Ministerul Educaţiei Naţionale Universitatea Maritimă Constanţa

Fundaţia Traian Lalescu

Concursul Naţional Studenţesc de Matematică “Traian Lalescu”

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BAREM - Secţiunea D

<u>Problema 1</u>

Metoda I:

Oficiu……………………………………………………………………………………..1p.

Aplicând transformarea Laplace, sistemul diferențial se transformă în sistemul algebric:

$$s^2X_k - sk - \frac{1}{2}(n-1)^{\frac{3}{2}} = X_1 + \ldots + X_{k-1} + X_{k+1} + \ldots + X_n,\ k = \overline{1,n}$$ ….……..2p.

Calculul lui Δ si al lui Δ_{X_k}………………....……………………………………………..…3p.

Soluţia sistemului algebric $X_k(s)$…….…………………………………………………..…1p.

Soluţia problemei Cauchy $x_k(t) = \left(k - \frac{n+1}{2}\right)\cos t + \frac{n}{2}e^{\sqrt{n-1}\,t} + \frac{1}{2}e^{-\sqrt{n-1}\,t}, k = \overline{1,n}$….3p.

Metoda II:

Oficiu……………………………………………………………………………………..1p.

Se notează $y(t) = x_1(t) + x_2(t) + \cdots + x_n(t)$ şi obţinem problema Cauchy

$y'' - (n-1)y = 0$ cu $y(0) = \frac{n(n+1)}{2},\ y'(0) = \frac{1}{2}n(n-1)^{\frac{3}{2}}$……………………………..2p.

Soluţia problemei Cauchy $y(t) = \frac{n}{2}e^{-\sqrt{n-1}t} + \frac{n^2}{2}e^{\sqrt{n-1}t}$…………………………………..2p.

Înlocuind în sistemul initial, acesta devine $x_k'' + x_k = y(t), k = \overline{1,n}$ ……………………..1p.

Rezolvarea sistemului (folosind Laplace sau ca ecuaţie liniară)……………………………4p.

<u>Problema 2</u>

Oficiu ……………………………………………………………………………….….1p.

a) $\dfrac{(z+1)^n}{z^{k+1}} = \dfrac{C_n^0 + C_n^1 z + \ldots + C_n^n z^n}{z^{k+1}}$ ……………………………………………………..1p.

$Rez(f,0) = C_n^k$……………………………………………………………………....….0.5p.

$I = 2\pi i C_n^k$ ……………………………………………………………………………………..0.5p.

b) $S = \sum_{n=0}^{\infty} \frac{1}{5^n} C_{2n}^n = \sum_{n=0}^{\infty} \frac{1}{5^n} \cdot \frac{1}{2\pi i} \cdot \int_{|z|=1} \frac{(z+1)^{2n}}{z^{n+1}} dz$ ……………………………………....…2p.

$\sum_{n=0}^{\infty} \left[\frac{(z+1)^2}{5z} \right]^n = \frac{5z}{-z^2+3z-1}, \quad pentru \quad \left| \frac{(z+1)^2}{5z} \right| < 1$ ……………………………………...2p.

$S = \frac{5}{2\pi i} \int_{|z|=1} \frac{1}{-z^2+3z-1} dz$ ………………………………………………………….….1p.

$S = 5\ Rez\left(f, \frac{3-\sqrt{5}}{2}\right)$…………………………………………………………………………..1p.

$S = \sqrt{5}$………………………………………………………………………………..1p.

Problema 3

Oficiu………………………………………………………………………………………1p.

Alege $f(t) = a(b - |t|), |t| \leq b$……………………………………………………………1p.

Calculează $\hat{f}(\omega) = \frac{4a}{\omega^2} \cdot sin^2 \frac{\omega b}{2}$………………………………………………………...3p.

$\mathcal{F}\left[\hat{f}(\omega)\right] = 2\pi f(-\omega)$……………………………………………………………………...1p.

$\hat{g}(\omega) = \frac{4 \cdot \left(\frac{1}{4}\right) sin^2\left(\frac{3t}{2}\right)}{t^2}$…………………………………………………………………….1p.

Finalizare $f(t) = \frac{\pi}{2}(3 - |t|), |t| < 3$…………………………………………………..3p.

Obs. S-a considerat $\hat{f}(\omega) = \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$

Problema 4

Oficiu…………………………………………………………………………………………1p.

$a)\ y'(t) = \frac{cos2\sqrt{t}}{\sqrt{t}}; y"(t) = -\frac{sin2\sqrt{t}}{t} - \frac{1}{2\sqrt{t}} cos2\sqrt{t}$…………………………………………0,5p.

$\Rightarrow f(t)$ identic 0$\Rightarrow$ $F = 0$ dacă nu ar exista condiţia $y'(0+)$ finită…………………………..0,5p.

b) F constantă $\Rightarrow F(s)$ identic 0 $\Rightarrow \lim_{s \to \infty} F(s) = 0 \Rightarrow y(0) = 0$……………………………....0,5p.

Deducerea ecuţiei $\frac{Y'(s)}{Y(s)} = -\frac{3}{2s} + \frac{1}{s^2}$…………………………………………………….....2,5p.

$Y(s) = c \cdot \frac{1}{s^{3/2}} \cdot e^{-\frac{1}{s}}$…………………………………………………………………………1p.

$Y(s) = c\sum_{n=0}^{\infty}(-1)^n \frac{1}{s^{n+\frac{3}{2}}} \cdot \frac{1}{n!}$..1p.

Aplicarea teoremei lui Heaviside:

$y(t) = c\sum_{n=0}^{\infty}(-1)^n \frac{t^{n+\frac{1}{2}}}{\Gamma\left(n+\frac{3}{2}\right)} \cdot \frac{1}{n!}$..1p.

$\Gamma\left(n+\frac{3}{2}\right) = \frac{(2n+1)!}{2^{2n+1}}\sqrt{\pi} \Longrightarrow y(t) = \frac{c}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n(2\sqrt{t})^{2n+1}}{(2n+1)!}$..1p.

$y(t) = \frac{c}{\sqrt{\pi}}\sin(2\sqrt{t})$..1p.