Ministerul Educaţiei Naţionale   Universitatea Maritimă Constanţa

Fundaţia Traian Lalescu

Concursul Naţional Studenţesc de Matematică "Traian Lalescu"

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BAREM - Secţiunea E

Problema 1

Oficiu ……………………………………………………………………………………1p.

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=2e^x\cos y \Rightarrow 2\frac{\partial u}{\partial x}=2e^x\cos y \Rightarrow \frac{\partial u}{\partial x}=e^x\cos y$ ……………………………………1p.

$u(x,y)=e^x\cos y+\Phi(y)$ ……………………………………………………………1p.

$\Delta u=0 \Rightarrow \Phi''(y)=0 \Rightarrow \Phi(y)=C_1y+C_2$ ……………………………………………2p.

$u(x,y)=e^x\cos y+C_1y+C_2$

$f'(z)=\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}=e^x\cos y-i(-e^x\sin y+C_1)$

$y=0 \Rightarrow f'(x)=e^x-iC_1 \Rightarrow f(x)=e^x-iC_1x+C_3$

$f(z)=e^z-iC_1z+C_3$ ……………………………………………………………………5p.

Problema 2

Oficiu……………………………………………………………………………………1p.

a) $D=\mathbb{C}\setminus\{0;\ \pm i\sqrt{2}; 2i; -3i\}$……………………………………………………0,5p.

$z=0$ esenţial

$z=\pm i\sqrt{2}$ poli de ordin 5

$z=2i$ pol de ordin 1, $z=-3i$ pol de ordin 1

Punctul de la ∞ este punct ordinar………………………………………………………1p.

b) $I=2\pi i\left(Rez(f,0)+Rez\left(f,i\sqrt{2}\right)+Rez\left(f,-i\sqrt{2}\right)\right)+\pi i Rez(f,2i)$……………………0,5p.

$I = -2\pi i\big(Rez(f,\infty) + Rez(f,-3i)\big) - \pi i Rez(f,2i)$..1p.

$Rez(f,\infty) = -3\pi i$..3p.

$Rez(f,3i) = 0$..1p.

$Rez(f,2i) = \frac{2^7}{5i}$...1p.

$I = -\frac{2\pi}{5}(64 + 3\pi)$...1p.

Problema 3

Oficiu ....1p.

$e^t * t^3[\sin t]^{(n)}$ ....1p.

$\frac{1}{s-1}(-1)^3\left(L[\sin t]^{(n)}(s)\right)^{(3)}$ ....1p.

$(\sin t)^{(n)} = \sin\left(t + \frac{n\pi}{2}\right)$...3p.

$L\left[\sin\left(t + \frac{n\pi}{2}\right)\right](s) = \frac{1}{s^2+1}\cdot\left(\cos\frac{n\pi}{2} + s\cdot\sin\frac{n\pi}{2}\right)$....................................3p.

Finalizare ....1p.

Problema 4

a) Oficiu...1p.

Metoda 1

$e^{ix} = z; cosx = \frac{z^2+1}{2z}$..1p.

$f(x) = \ln(1 - az) + ln\left(1 - \frac{a}{z}\right)$...2p.

$\ln(1 - az) = -\sum_{n=1}^{\infty}\frac{a^n z^n}{n}$...0,5p.

$|az| < 1$

$\ln\left(1 - \frac{a}{z}\right) = -\sum_{n=1}^{\infty}\frac{a^n}{n\cdot z^n}$..0,5p.

$f(x) = -2\sum_{n=1}^{\infty} \frac{a^n}{n} \cos(nx)$……………………………………………………………….….2p.

Metoda 2

$g = f'(x)$……………………………………………………………………………...…0,5p.

$a_n + ib_n \overset{e^{ix}=z}{=} -\frac{a}{\pi} \int_{|z|=1} \frac{(z^2-1)z^{n-1}}{az^2 - z(a^2+1) + a} dz$……………………………………………………...….2p.

$b_n = -2a^n, a_n = 0, n \geq 1, a_0 = 0$……………………………………………………..2,5p.

$f(x) = -2\sum_{n=1}^{\infty} \frac{a^n}{n} \cos(nx)$……………………………………………………………...1p.

b) $x = 0$…………………………………………………………………………………1p.

$a = \frac{1}{2}$…………………………………………………………………………………….1p.

Finalizare: $\sum_{n=1}^{\infty} \frac{1}{n2^n} = ln2$………………………………………………………..…1p.